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Analysis of Covariance with Linear Regression Error Model on Antenna Control Unit Tracking

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14. ABSTRACT
Over the past several years DoD has imposed constraints on test deliverables, requiring objective measures of test objective results, i.e., statistically defensible test and evaluation (SDT&E) methods and results. These constraints force tester to employ statistical hypotheses, analyses and perhaps modeling to assess test results objectively, i.e., on statistical metrics, probability of confidence, logical inference to supplement technical expertise, rather than rely solely on expertise, which is subjective. In this paper we apply linear regression modeling and analysis of variance (ANOVA) to decide for the null- or alternative-hypotheses of a telemetry antenna control unit's (ACU) ability to track on C-band while receiving S- or C-band data. Together, regression and ANOVA compose a method known as analysis of covariance (ANCOVA). The intent is to present examples of tools and techniques useful for SDT&E methodologies in testing.

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ANALYSIS OF COVARIANCE WITH LINEAR REGRESSION ERROR MODEL ON ANTENNA CONTROL UNIT TRACKING

By
DANIEL T. LAIRD

ABSTRACT

Over the past several years DoD imposed constraints on test deliverables, requiring objective measures of test results, i.e., statistically defensible test and evaluation (SDT&E) methods and results. These constraints force the tester to employ statistical hypotheses, analyses and perhaps modeling to assess test results objectively, i.e., based on statistical metrics, probability of confidence and logical inference to supplement rather than rely solely on expertise, which is too subjective. Experts often disagree on interpretation. Numbers, although interpretable, are less variable than opinion. Logic, statistical inference and belief are the bases of testable, repeatable and refutable hypothesis and analyses. In this paper we apply *linear regression modeling* and *analysis of variance* (ANOVA) to time-space position information (TSPI) to determine if a telemetry (TM) antenna control unit (ACU) under test (AUT) tracks statistically, thus as efficiently, in C-band while receiving both C- and S-band signals. Together, regression and ANOVA compose a method known as *analysis of covariance* (ANCOVA). In this, the second of three papers, we use data from a range test, but make no reference to the systems under test, nor to causes of error. The intent is to present examples of tools and techniques useful for SDT&E methodologies in testing.

KEY TERMS

Null- and alternative-hypotheses, tracking mode, TM, AGC, ACU, Scan rate, Slew rate, TSPI samples, GPS, INS, observables, calculated, dataframe, R, inner-product, modeling, ANOVA, ANCOVA, *F*-test and *t*-test, PDF, CDF.

INTRODUCTION

Using statistical methods we decide to either reject or accept the *null-hypothesis*, H_0 or its *alternative hypothesis*, H_1. We do so based on objective criteria derived from analyses. The test null hypothesis is:

H_0: *AUT tracking errors are statistically identical independent of data carrier*.

The AUT tracks on conically scanned[a] signal strength, or amplitude sampled from *automatic gain control* (AGC) circuits, and on controllable conic-scan (*Scan*) and antenna slew (*Slew*) rates. The test includes S-band and C-band data channels (which carry TSPI) of a C-band tracking AUT. Ideally the tracking algorithm should have no dependence on observed angles or data band, i.e., the tracking algorithm should not depend on pointing angle, as the conic-scan center is controlled to stay on peak tracking band signal strength. Of course this is ideal, and we've discovered that *tracking-error* is dependent on combinations of both azimuth and

[a] Rf 3.

elevation, and uniquely so receiving in C- or S-band. We had no information about the ACU tracking algorithm, only commanded and observed data, which requires another source to determine tracking error.[b] Data includes observed and commanded angles, AGC levels and the rate control states. This paper's focus is analysis of *tracking-error* using TSPI data culled from each of the carriers at specific test points. In a companion paper[c] we extend analyses into synthesis of an *auto-tracking* model independent of observed angle. All modeling and analyses is in R, a statistical application.

ANALYSIS OF COVARIANCE

For modeling purposes, received TM signal power is a bounded continuous parameter sampled from receiver automatic gain control (AGC) circuits of an integrated AUT. Local azimuth and elevation angles are also modeled as bounded continuous parameters, commanded by and measured from the AUT feedback. An ACU employs two categorical *tangential*[d] or *rate* control parameters, designated as *Scan* and *Slew*. Other available data include target TSPI: global position system (GPS) and inertial navigation system (INS) data. Since carriers radiate from a moving target that changes in both global position and attitude, we experimented with various regression models with parameters from the target. Due to length constraints we cannot discuss all models and variations. Thus, we focus on a single tracking model based on a subset of ACU parameters and target TSPI (GPS and INS) samples. Target TSPI is required for calculating measures on tracking accuracy of milli-radian precision samples to build a tracking error model. ANOVA reveals numerical estimates of the target's attitude on *tracking-error*. We construct a *tracking-error* model of a continuous *expected error*, linearly regressed against track angles, AGC, rate controls and a subset of target attitude. Aircraft attitude *yaw*, *pitch* and *roll* were investigated as potential sources of transmitter radiation pattern fluctuation on receiver AGC levels. ANCOVA employed a final *reduced* tracking-error model. Paper length limitations allow only brief discussion of *full* model reduction.

LINEAR REGRESSON MODEL

The auto-tracking model is built on a *linear predictor* of a regressed measure vector and its conjugate observable-control space, which together form an inner-product space[e]. The predictor is a *functional form* specifically an *inner-product*:

$$\boldsymbol{\beta}\bullet\mathbf{x} = \beta_0 + \Sigma\ \beta_k x_k \equiv bx. \qquad (1)$$

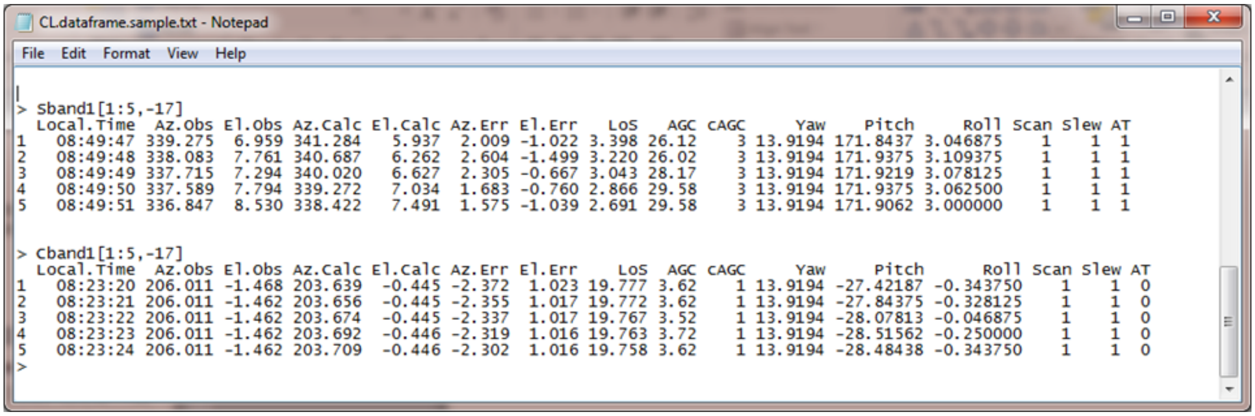
The bx is a common statistical symbol for the inner-product. $\boldsymbol{\beta}$ designates predictor coefficients, conjugates regressed against the sampled independent data variables $\mathbf{x}$. β_0 designates the predictor intercept (the *null-model*). The response is a continuous variable predicted by bx. We designate the model:

$$lm(y \sim bx). \qquad (2)$$

where y designates estimated error. Parameters are collated as a *dataframe* constructed from sample files. A sample is shown in Figure 1. We see from the figure a separation of dataframes per receiver band. The dataframe contains time common to both ACU and TSPI. Time

[b] Rf. 1.
[c] Rf. 2.
[d] Rf. 1, 7 & 8.
[e] Rf. 8.

alignment correlates ACU and TSPI samples at 1Hz. Observed angles and AGC levels come from the ACU, the expected angles and line-of-sight (LoS) are derived via a *geometric model*[f] that translates TSPI global position to local azimuth and elevation angles of LoS between tracker and target. Rate control is a state from the ACU. AT is derived from the ACU elevation and azimuth tracking *modes*. Attitude data is sampled from target INS. Two categorical tangential[g] control parameters are conical-scan rate, *Scan* $\in \{2\}^+$ and four-level slew rate, *Slew* $\in \{4\}^{+}$[h].



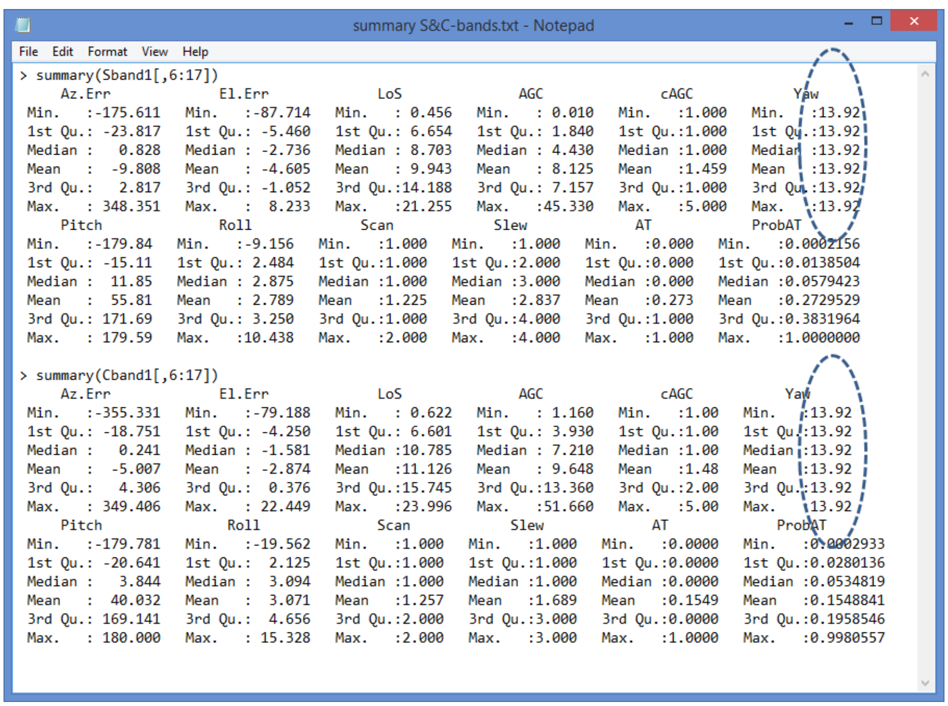
```
CL.dataframe.sample.txt - Notepad
File  Edit  Format  View  Help

> Sband1[1:5,-17]
  Local.Time  Az.Obs El.Obs Az.Calc El.Calc Az.Err El.Err   LoS   AGC CAGC    Yaw    Pitch     Roll Scan Slew AT
1   08:49:47 339.275  6.959 341.284   5.937  2.009 -1.022 3.398 26.12    3 13.9194 171.8437 3.046875    1    1  1
2   08:49:48 338.083  7.761 340.687   6.262  2.604 -1.499 3.220 26.02    3 13.9194 171.9375 3.109375    1    1  1
3   08:49:49 337.715  7.294 340.020   6.627  2.305 -0.667 3.043 28.17    3 13.9194 171.9219 3.078125    1    1  1
4   08:49:50 337.589  7.794 339.272   7.034  1.683 -0.760 2.866 29.58    3 13.9194 171.9375 3.062500    1    1  1
5   08:49:51 336.847  8.530 338.422   7.491  1.575 -1.039 2.691 29.58    3 13.9194 171.9062 3.000000    1    1  1

> Cband1[1:5,-17]
  Local.Time  Az.Obs El.Obs Az.Calc El.Calc Az.Err El.Err    LoS  AGC CAGC    Yaw     Pitch      Roll Scan Slew AT
1   08:23:20 206.011 -1.468 203.639  -0.445 -2.372  1.023 19.777 3.62    1 13.9194 -27.42187 -0.343750    1    1  0
2   08:23:21 206.011 -1.462 203.656  -0.445 -2.355  1.017 19.772 3.62    1 13.9194 -27.84375 -0.328125    1    1  0
3   08:23:22 206.011 -1.462 203.674  -0.445 -2.337  1.017 19.767 3.52    1 13.9194 -28.07813 -0.046875    1    1  0
4   08:23:23 206.011 -1.462 203.692  -0.446 -2.319  1.016 19.763 3.72    1 13.9194 -28.51562 -0.250000    1    1  0
5   08:23:24 206.011 -1.462 203.709  -0.446 -2.302  1.016 19.758 3.62    1 13.9194 -28.48438 -0.343750    1    1  0
>
```

Figure 1 Samples Dataframe

All categorical parameters are *typed* as *factors* for ANOVA. Angles are bounded modulo 360º: {Az.Obs, El.Obs, Yaw, Pitch, Roll} ⊂ [0, 360].



```
summary S&C-bands.txt - Notepad
File  Edit  Format  View  Help
> summary(Sband1[,6:17])
     Az.Err              El.Err              LoS               AGC               cAGC             Yaw
 Min.   :-175.611   Min.   :-87.714   Min.   : 0.456   Min.   : 0.010   Min.   :1.000   Min.   :13.92
 1st Qu.: -23.817   1st Qu.: -5.460   1st Qu.: 6.654   1st Qu.: 1.840   1st Qu.:1.000   1st Qu.:13.92
 Median :   0.828   Median : -2.736   Median : 8.703   Median : 4.430   Median :1.000   Median :13.92
 Mean   :  -9.808   Mean   : -4.605   Mean   : 9.943   Mean   : 8.125   Mean   :1.459   Mean   :13.92
 3rd Qu.:   2.817   3rd Qu.: -1.052   3rd Qu.:14.188   3rd Qu.: 7.157   3rd Qu.:1.000   3rd Qu.:13.92
 Max.   : 348.351   Max.   :  8.233   Max.   :21.255   Max.   :45.330   Max.   :5.000   Max.   :13.92
     Pitch              Roll             Scan            Slew             AT             ProbAT
 Min.   :-179.84   Min.   :-9.156   Min.   :1.000   Min.   :1.000   Min.   :0.000   Min.   :0.0002156
 1st Qu.: -15.11   1st Qu.: 2.484   1st Qu.:1.000   1st Qu.:2.000   1st Qu.:0.000   1st Qu.:0.0138504
 Median :  11.85   Median : 2.875   Median :1.000   Median :3.000   Median :0.000   Median :0.0579423
 Mean   :  55.81   Mean   : 2.789   Mean   :1.225   Mean   :2.837   Mean   :0.273   Mean   :0.2729529
 3rd Qu.: 171.69   3rd Qu.: 3.250   3rd Qu.:1.000   3rd Qu.:4.000   3rd Qu.:1.000   3rd Qu.:0.3831964
 Max.   : 179.59   Max.   :10.438   Max.   :2.000   Max.   :4.000   Max.   :1.000   Max.   :1.0000000

> summary(Cband1[,6:17])
     Az.Err              El.Err              LoS               AGC               cAGC           Yaw
 Min.   :-355.331   Min.   :-79.188   Min.   : 0.622   Min.   : 1.160   Min.   :1.00   Min.   :13.92
 1st Qu.: -18.751   1st Qu.: -4.250   1st Qu.: 6.601   1st Qu.: 3.930   1st Qu.:1.00   1st Qu.:13.92
 Median :   0.241   Median : -1.581   Median :10.785   Median : 7.210   Median :1.00   Median :13.92
 Mean   :  -5.007   Mean   : -2.874   Mean   :11.126   Mean   : 9.648   Mean   :1.48   Mean   :13.92
 3rd Qu.:   4.306   3rd Qu.:  0.376   3rd Qu.:15.745   3rd Qu.:13.360   3rd Qu.:2.00   3rd Qu.:13.92
 Max.   : 349.406   Max.   : 22.449   Max.   :23.996   Max.   :51.660   Max.   :5.00   Max.   :13.92
     Pitch               Roll               Scan            Slew             AT               ProbAT
 Min.   :-179.781   Min.   :-19.562   Min.   :1.000   Min.   :1.000   Min.   :0.0000   Min.   :0.0002933
 1st Qu.: -20.641   1st Qu.:  2.125   1st Qu.:1.000   1st Qu.:1.000   1st Qu.:0.0000   1st Qu.:0.0280136
 Median :   3.844   Median :  3.094   Median :1.000   Median :1.000   Median :0.0000   Median :0.0534819
 Mean   :  40.032   Mean   :  3.071   Mean   :1.257   Mean   :1.689   Mean   :0.1549   Mean   :0.1548841
 3rd Qu.: 169.141   3rd Qu.:  4.656   3rd Qu.:2.000   3rd Qu.:3.000   3rd Qu.:0.0000   3rd Qu.:0.1958546
 Max.   : 180.000   Max.   : 15.328   Max.   :2.000   Max.   :3.000   Max.   :1.0000   Max.   :0.9980557
```

Figure 2 Dataframe Summary

[f] Rf. 1.

[g] Rf. 7 & 8.

[h] $\{n\}^+ \equiv \{1, \ldots, n\}$; $\{n\} \equiv \{0, \ldots, n-1\}$.

Note the constant Yaw in the statistical summary of the data, figure 2. This will influence our final model parameterization. Automatic gain control is a derived parameter from multiple receiver status samples. Empirical decibel range samples are: $\{AGC\} \subset 0 \leq dB \leq 52$. We generalize linear models for both azimuth and elevation errors in R as:

lm(Az.Err ~ Az.Obs + El.Obs + (c)AGC + … + Pitch + Roll) (3a)

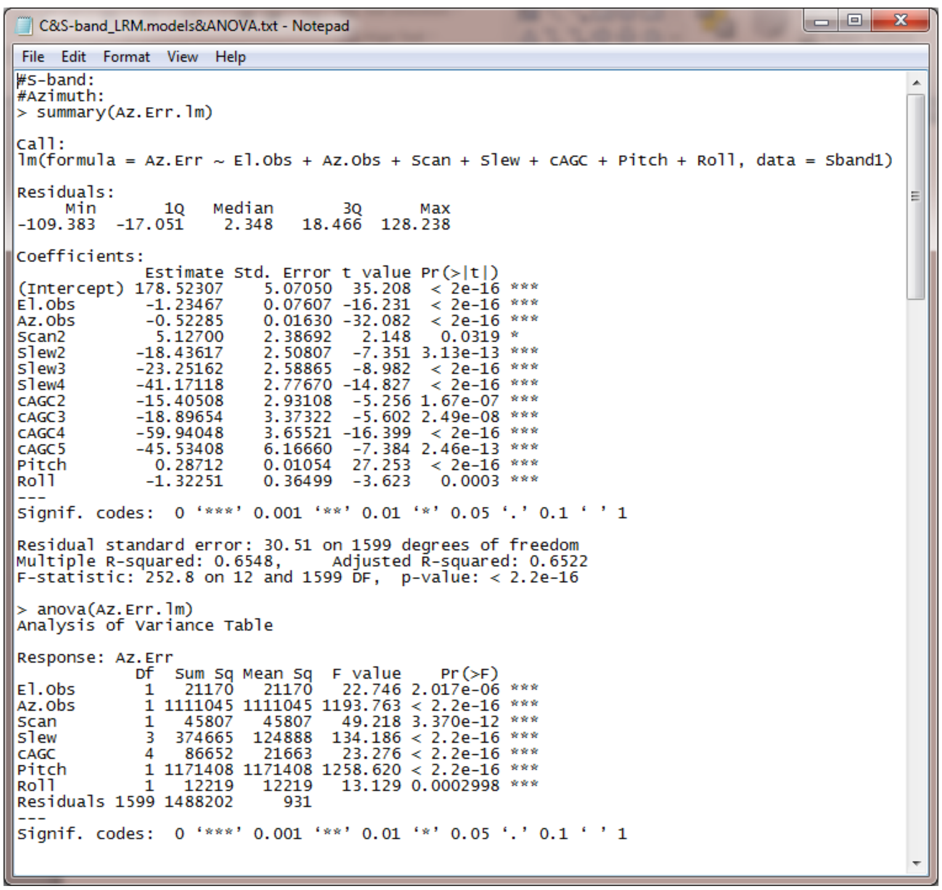
lm(El.Err ~ El.Obs + Az.Obs + (c)AGC + … + Pitch + Roll) (3b)

Parameters are particular to each model, unique for band and angle type (azimuth or elevation); thus we have four models to consider. The parenthesized 'c' in (3) indicates either continuous (AGC) or categorized gain control (cAGC) in the model. The details of each model are shown in figures 3a and 3b. The categorization of AGC $\rightarrow$ cAGC adds no significance to the model predictability, so will not play a part in the final model.

The final models are:

lm(Az.Err ~ Az.Obs + El.Obs + AGC + Slew + Scan + Pitch + Roll) (3c)

lm(El.Err ~ El.Obs + Az.Obs + AGC + Slew + Scan + Pitch + Roll) (3d)


C&S-band_LRM.models&ANOVA.txt - Notepad

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```
#S-band:
#Azimuth:
> summary(Az.Err.lm)

Call:
lm(formula = Az.Err ~ El.Obs + Az.Obs + Scan + Slew + cAGC + Pitch + Roll, data = Sband1)

Residuals:
     Min       1Q   Median       3Q      Max
-109.383  -17.051    2.348   18.466  128.238

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 178.52307    5.07050  35.208  < 2e-16 ***
El.Obs       -1.23467    0.07607 -16.231  < 2e-16 ***
Az.Obs       -0.52285    0.01630 -32.082  < 2e-16 ***
Scan2         5.12700    2.38692   2.148   0.0319 *
Slew2       -18.43617    2.50807  -7.351 3.13e-13 ***
Slew3       -23.25162    2.58865  -8.982  < 2e-16 ***
Slew4       -41.17118    2.77670 -14.827  < 2e-16 ***
cAGC2       -15.40508    2.93108  -5.256 1.67e-07 ***
cAGC3       -18.89654    3.37322  -5.602 2.49e-08 ***
cAGC4       -59.94048    3.65521 -16.399  < 2e-16 ***
cAGC5       -45.53408    6.16660  -7.384 2.46e-13 ***
Pitch         0.28712    0.01054  27.253  < 2e-16 ***
Roll         -1.32251    0.36499  -3.623   0.0003 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 30.51 on 1599 degrees of freedom
Multiple R-squared: 0.6548,     Adjusted R-squared: 0.6522
F-statistic: 252.8 on 12 and 1599 DF,  p-value: < 2.2e-16

> anova(Az.Err.lm)
Analysis of Variance Table

Response: Az.Err
            Df  Sum Sq Mean Sq  F value    Pr(>F)
El.Obs       1   21170   21170   22.746 2.017e-06 ***
Az.Obs       1 1111045 1111045 1193.763 < 2.2e-16 ***
Scan         1   45807   45807   49.218 3.370e-12 ***
Slew         3  374665  124888  134.186 < 2.2e-16 ***
cAGC         4   86652   21663   23.276 < 2.2e-16 ***
Pitch        1 1171408 1171408 1258.620 < 2.2e-16 ***
Roll         1   12219   12219   13.129 0.0002998 ***
Residuals 1599 1488202     931
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Figure 3a Azimuth Model Form and ANOVA Results

We see both antenna observed angles and target attitude contribute significantly to error estimation. This makes intuitive sense since the target radiation pattern is moving. Attitude *yaw* was eliminated by the model as an insignificant constant. This was obvious via the statistical

summary of the dataframes (Figure 2 above). We assume yaw was not updating the acquisition store for unknown causes.

C&S-band_LRM.models&ANOVA.txt - Notepad

File Edit Format View Help

```
#Elevation:
> summary(El.Err.lm)

Call:
lm(formula = El.Err ~ El.Obs + Az.Obs + cAGC + Pitch + Roll, data = Sband1)

Residuals:
    Min      1Q  Median      3Q     Max 
-18.497  -1.361  -0.451   1.074  32.496 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) -2.430322   0.543233  -4.474 8.22e-06 ***
El.Obs      -0.716422   0.007913 -90.536  < 2e-16 ***
Az.Obs       0.015056   0.001600   9.412  < 2e-16 ***
cAGC2        3.944317   0.327669  12.038  < 2e-16 ***
cAGC3        4.065816   0.376666  10.794  < 2e-16 ***
cAGC4        8.494049   0.408506  20.793  < 2e-16 ***
cAGC5       15.958711   0.689453  23.147  < 2e-16 ***
Pitch       -0.007295   0.001180  -6.182 8.00e-10 ***
Roll        -0.069751   0.040550  -1.720   0.0856 .  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 3.428 on 1603 degrees of freedom
Multiple R-squared: 0.8722,     Adjusted R-squared: 0.8716 
F-statistic:  1368 on 8 and 1603 DF,  p-value: < 2.2e-16

> anova(El.Err.lm)
Analysis of Variance Table

Response: El.Err
            Df Sum Sq Mean Sq   F value    Pr(>F)    
El.Obs       1 117002  117002 9957.6424 < 2.2e-16 ***
Az.Obs       1     44      44    3.7423   0.05323 .  
cAGC         4  11005    2751  234.1471 < 2.2e-16 ***
Pitch        1    472     472   40.1954  2.98e-10 ***
Roll         1     35      35    2.9588   0.08560 .  
Residuals 1603  18835      12                        
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Figure 3b Elevation Model Form and ANOVA Results

The regression coefficients and statistics for our tracking-error model tracking in azimuth and elevation in C-band while receiving data from S- or C-band carriers are shown in Figures 3a and b. We see that the ‘best fit model’ is slightly different for the angles. Note that categorizing the AGC (cAGC) for S-band samples yields a slightly better model, based on the R-squared value on less degrees of freedom. Partitioning AGC from a single to multiple continuous levels reduces degrees of freedom in the model. We often reduce models by eliminating or recategorizing parameters with a high *p*-value, or what amounts to less information and prediction power. These *p*-values are not the probability of error, but results of a *t*-test on the probability that the modeled parameter coefficient contributes to informative model results, e.g., is merely a random variable with no more explanatory or predictive power than the *empty* or *null-model*, i.e., the model based only on dispersion or deviance around the mean of observation (the model intercept, β_0). It is convenient for explanatory purposes that the full C-band error model is not regressed against categorized AGC, as doing so revealed no explanatory advantage. So we kept AGC as sole continuous parameter.

C&S-band_LRM.models&ANOVA.txt - Notepad

File Edit Format View Help

```
#C-band:
#Azimuth:
> summary(Az.Err.lm)

Call:
lm(formula = Az.Err ~ El.Obs + Az.Obs + Scan + Slew + AGC + Pitch + Roll, data = Cband1)

Residuals:
     Min       1Q   Median       3Q      Max
-306.539  -26.644    7.237   28.349  156.808

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) 196.499256   3.788187  51.872  < 2e-16 ***
El.Obs       -1.436565   0.093676 -15.335  < 2e-16 ***
Az.Obs       -0.685692   0.012254 -55.957  < 2e-16 ***
Scan2       -19.483582   2.576527  -7.562 5.08e-14 ***
Slew2         8.706860   2.564803   3.395 0.000695 ***
Slew3        15.410312   2.571515   5.993 2.28e-09 ***
AGC           0.842245   0.104999   8.021 1.42e-15 ***
Pitch         0.185734   0.009024  20.582  < 2e-16 ***
Roll         -1.798591   0.222791  -8.073 9.42e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 46.6 on 3400 degrees of freedom
Multiple R-squared: 0.5253,     Adjusted R-squared: 0.5242
F-statistic: 470.4 on 8 and 3400 DF,  p-value: < 2.2e-16

> anova(Az.Err.lm)
Analysis of Variance Table

Response: Az.Err
            Df  Sum Sq Mean Sq   F value    Pr(>F)
El.Obs       1    6780    6780    3.1225   0.07731 .
Az.Obs       1 6325504 6325504 2913.2558 < 2.2e-16 ***
Scan         1   45639   45639   21.0191 4.711e-06 ***
Slew         2  107066   53533   24.6551 2.341e-11 ***
AGC          1  116624  116624   53.7118 2.886e-13 ***
Pitch        1 1427786 1427786  657.5767 < 2.2e-16 ***
Roll         1  141510  141510   65.1735 9.424e-16 ***
Residuals 3400 7382364    2171
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

#Elevation:
> summary(El.Err.lm)

Call:
lm(formula = El.Err ~ El.Obs + Slew + AGC + Pitch + Roll, data = Cband1)

Residuals:
     Min       1Q   Median       3Q      Max
-22.3262  -2.6172  -0.8927   2.1406  26.5152

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.4452350  0.1833150   7.884 4.24e-15 ***
El.Obs      -0.6859190  0.0089626 -76.531  < 2e-16 ***
Slew2        0.9252144  0.2194964   4.215 2.56e-05 ***
Slew3       -0.0811202  0.1878749  -0.432    0.666
AGC          0.1898839  0.0098553  19.267  < 2e-16 ***
Pitch       -0.0046350  0.0008678  -5.341 9.84e-08 ***
Roll         0.0856879  0.0216048   3.966 7.45e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.542 on 3402 degrees of freedom
Multiple R-squared: 0.6593,     Adjusted R-squared: 0.6587
F-statistic:  1097 on 6 and 3402 DF,  p-value: < 2.2e-16

> anova(El.Err.lm)
Analysis of Variance Table

Response: El.Err
            Df Sum Sq Mean Sq   F value    Pr(>F)
El.Obs       1 126565  126565 6135.2197 < 2.2e-16 ***
Slew         2    282     141    6.8415  0.001083 **
AGC          1   7443    7443  360.7852 < 2.2e-16 ***
Pitch        1   1198    1198   58.0843 3.236e-14 ***
Roll         1    325     325   15.7303 7.455e-05 ***
Residuals 3402  70180      21
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Figure 4a Azimuth and Elevation
Regression Coefficients and Statistics

We also discovered via regression and ANOVA that *Slew* factor levels are statistically insignificant for elevation over both bands. Therefore, azimuth and elevation have statistically different models, even within a set signal band. Note that each angle model has very different predictor coefficients across bands, indicating the parameters influence in tracking are distinct to

band. The regression coefficients and statistics for C-band tracking-error model are shown in Figure 4. Regression summary and ANOVA tables reveal that S-band elevation is the most difficult of the angle models to fit. All *t*-test (Pr(>| t |) in the summary) show statistical significance for the respective parameters, and the ANOVA results confirm this; but note that the ANOVA *F*-test (Pr(>F) in summary) significance indicate a parameter significance much less than the regression *t*-test significance. We also see that Slew rate 3 has no statistical significance in regression on elevation in C-band. These ANCOVA results show our reduced parameters are sufficiently significant, enough so that we can employ this model to predict elevation error. Discussion with Air Force Test Center (AFTC) Telemetry Technical Expert confirmed elevation as the more difficult angle to track regardless of scanning and/or slewing[i]; thus we deduce it the more difficult to model, and our results bear this out. We also see that both antennas observed tracker-to-target angles and target attitude contribute significantly to error estimation.

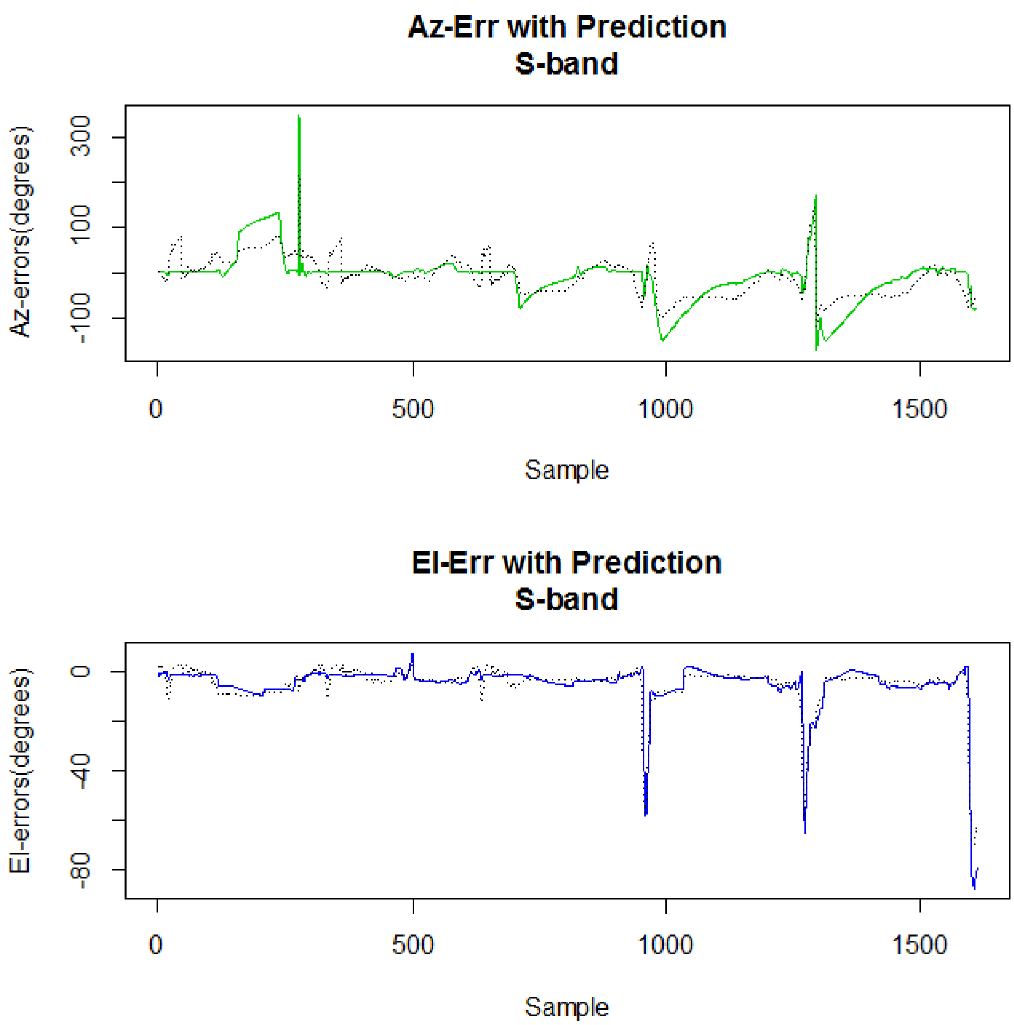

Figure 5a Azimuth and Elevation S-Band Samples:
Errors with Predictions (colored)

[i] Rf. 3.

The 'best model' fit employs continuous AGC for C-band samples, as this yields a slightly better estimation of the '*R-squared*' and '*Residual standard error*' values on less degrees of freedom. Interestingly, the R^2 value indicates that elevation is better explained w.r.t. the null-models than is azimuth for both S- and C-band. All values contribute probable explanatory and predictive significance, and the *F*-statistic indicates that model explains well the observed, i.e., measured angle errors.

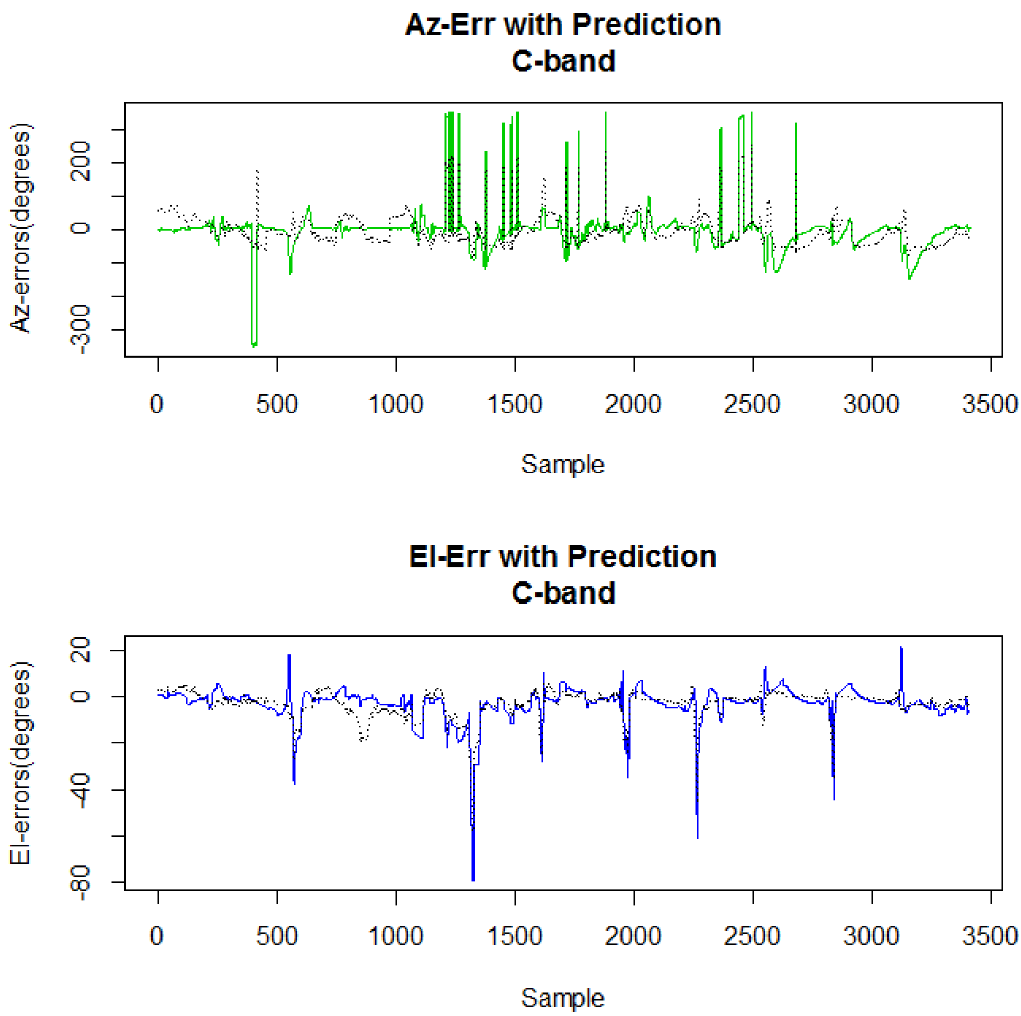

Figure 5b Azimuth and Elevation C-Band Samples:
Errors with Predictions (colored)

Observed and predicted error from the model are shown in Figures 5a and b for S-band and C-band received data, respectively. The track profile comparisons used observed angles and theoretical angles, the latter derived from a geometric model.[j] The colored curve designates the observed error w.r.t. the derived angle; the black is the predicted error based on the model

[j] Rf. 1.

parameters. It is clear from Figures 5 that the C-band error in azimuth has several large differences from the expected value. These errors were probably due to the planar slewing.

SUMMARY AND CONCLUSION

This paper focuses on analyzing tracking error building an observation error model and ANOVA to explore statistical sources of errors in measures, or observation. The ANOVA and model results agree, showing not only the tracking ACU and carrier, but the tracked target dynamics (attitudinal changes) contribute to measurement based on received signal strength and the state of control of the ACU. Our analysis and models indicate that we should reject the null hypothesis, H_0, that the AUT tracks identically, within a defined error tolerance over C-band for data received in S-band or C-band carriers and accept the alternative hypothesis:

H_1: *AUT tracks statistically differently in C-band while receiving S-band vs. C-band data*.

That there is an error in tracking is evidenced by the data, statistics and models will be developed further in the companion paper that focuses on synthesizing an auto-tracking model dependent on ACU, independent of both measured, or observed angles and target attitudinal dynamics. We also infer a practical difference, but make no hypothesis as to cause(s). We investigate this further in the third of three companion papers – an investigation of a logistic regression on autotrack.

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